Rossmoyne Senior High School

MATHEMATICS SPECIALIST UNIT 3

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet.

3.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that you do not use pencil, except in diagrams.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

         SN085-114-2

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

Two complex numbers are $u = \sqrt{3} + i$ and $v = 3 \operatorname{cis}\left(-\dfrac{\pi}{4}\right)$.

(a) Determine the argument of uv. (2 marks)

(b) Simplify $|v \times \bar{v} \times u^{-1}|$. (2 marks)

(c) Determine z in polar form if $5zu = v^2$. (2 marks)

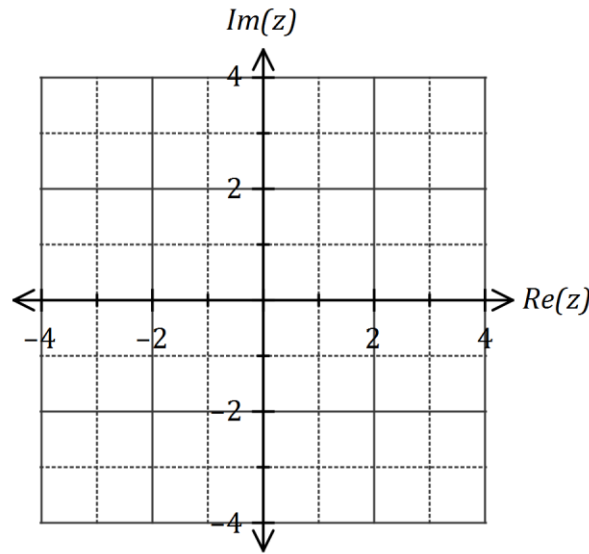**Question 10**   **(5 marks)**

(a)   The vector equation of a curve is given by $\mathbf{r}(\mu) = (\mu^2 + 3)\mathbf{i} + (\mu - 5)\mathbf{j}$.
         Determine the corresponding **Cartesian** equation for the curve.   (2 marks)

(b)   A sphere has Cartesian equation $x^2 + y^2 + z^2 + 8x - 12y + 2z = 0$.
         Determine the **vector** equation of the sphere.   (3 marks)
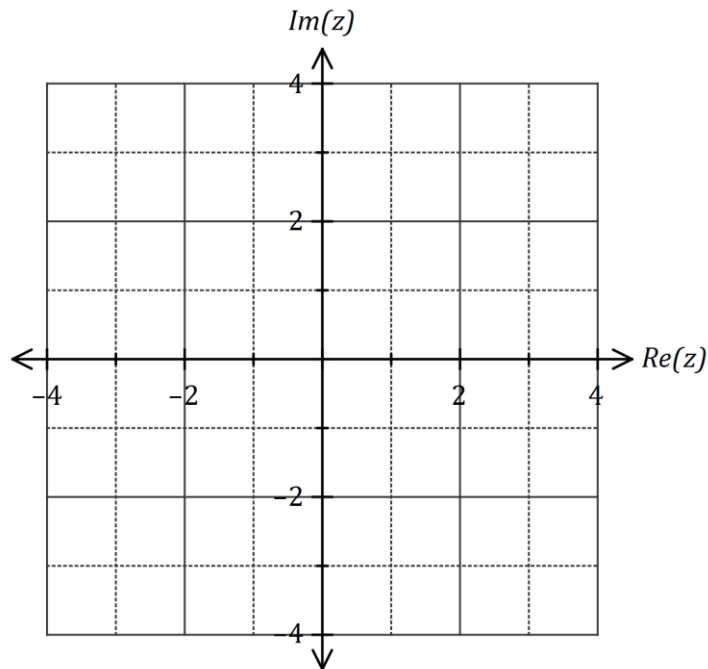
Question 11   **(9 marks)**

(a)   On the Argand plane below, sketch the locus of $|z - 1 - i| = |z + 1 - 3i|$, where z is a
        complex number.   (3 marks)

(b)   Consider the three inequalities $|z + 2 + 2i| \leq 2$, $\arg(z) \geq -\dfrac{3\pi}{4}$ and $\mathrm{Re}(z) \leq -1$.

        (i)   On the Argand plane below, shade the region that represents the complex
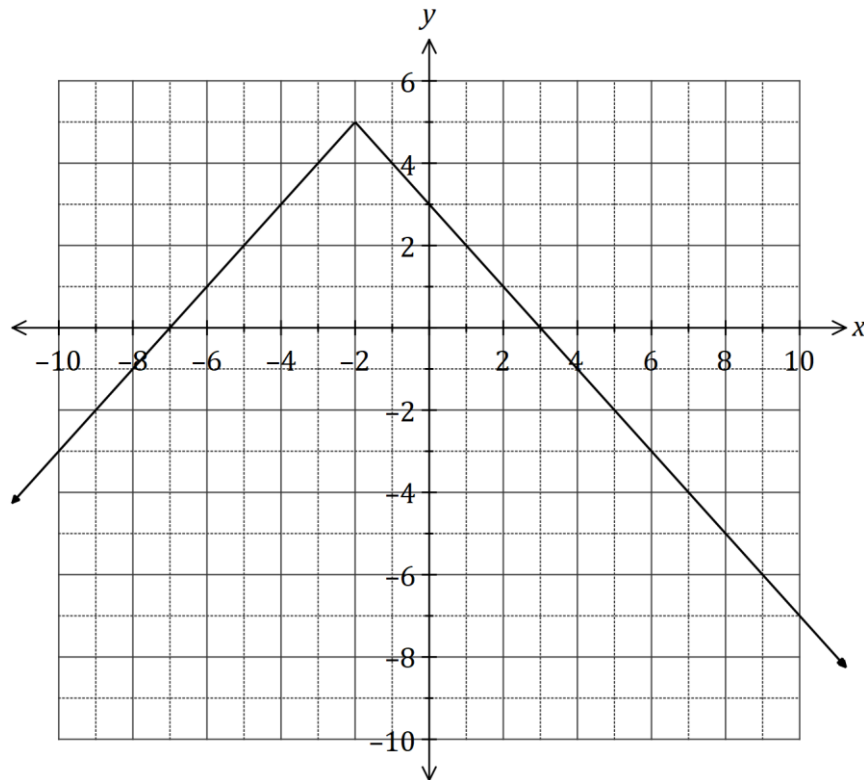   numbers satisfying **all** of these inequalities.   (5 marks)

        (ii)   Determine the maximum possible value of $|z|$ within the shaded region.

                                                                                (1 mark)

Question 12   **(7 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a|x + b| + c$, where a, b and c are constants.

(a)   Add the graph of $y = g(x)$ to the axes above, where $g(x) = 3|x - 1| - 10$.   (2 marks)

(b)   Determine the values of a, b and c.   (3 marks)

(c)   Using your graph, or otherwise, solve $f(x) + g(x) = 0$.   (2 marks)

See next page

Question 13 **(7 marks)**

A particle, with initial velocity vector $(10, 3, -15)$ ms^{-1}, experiences a constant acceleration for 16 seconds. The velocity vector of the particle at the end of the 16 seconds is $(34, -37, -31)$ ms^{-1}.

(a) Determine the magnitude of the acceleration. (3 marks)

(b) Calculate the change in displacement of the particle over the 16 seconds. (4 marks)

Question 14   **(8 marks)**

The position vectors of bodies L and M at times λ and μ are given by

$$\mathbf{r}_L = 10\mathbf{i} - \mathbf{j} + 6\mathbf{k} + \lambda(-2\mathbf{i} + a\mathbf{j} + b\mathbf{k})$$

and

$$\mathbf{r}_M = 3\mathbf{i} - 11\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$$

where a and b are constants, times are in seconds and distances are in metres.

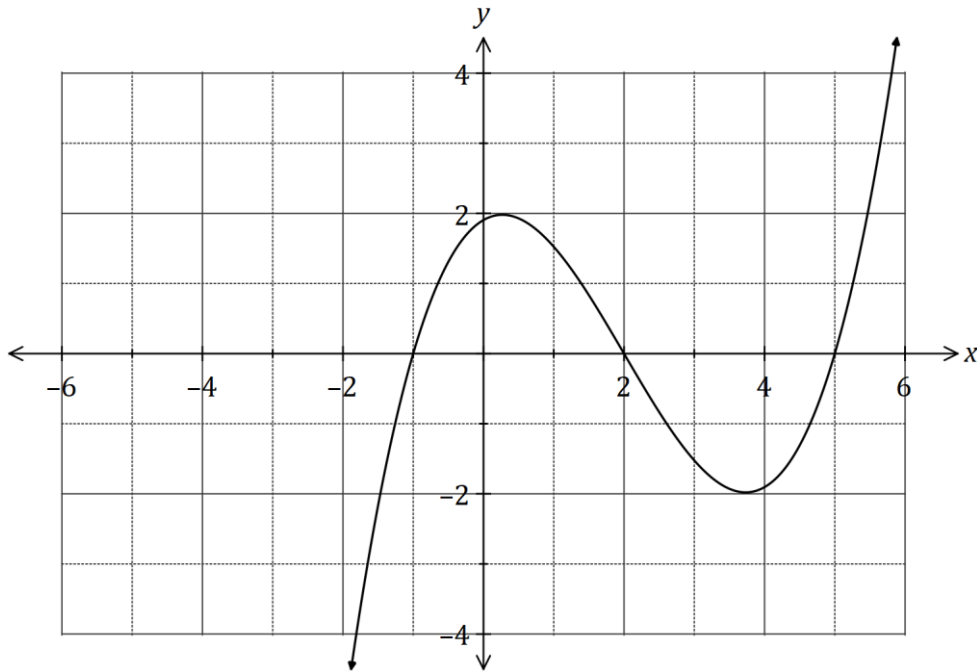(a)   Given that the paths of L and M intersect, show that $4a - b + 10 = 0$.   (4 marks)

(b)   Given that the paths of L and M are also perpendicular, determine the values of a and b, and the position vector of the point of intersection of the paths.   (4 marks)

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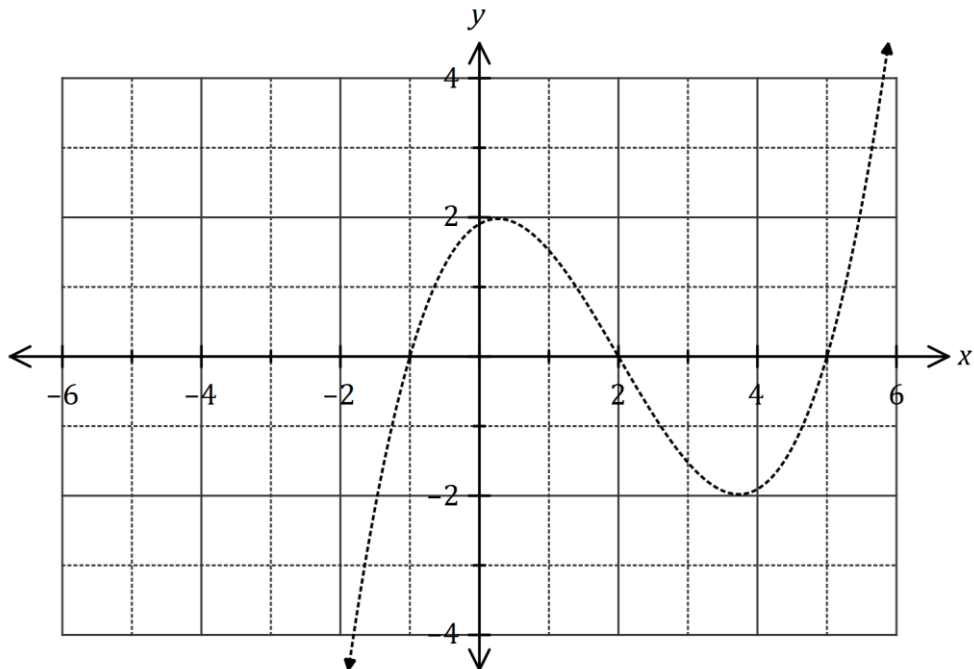
Question 15 **(8 marks)**

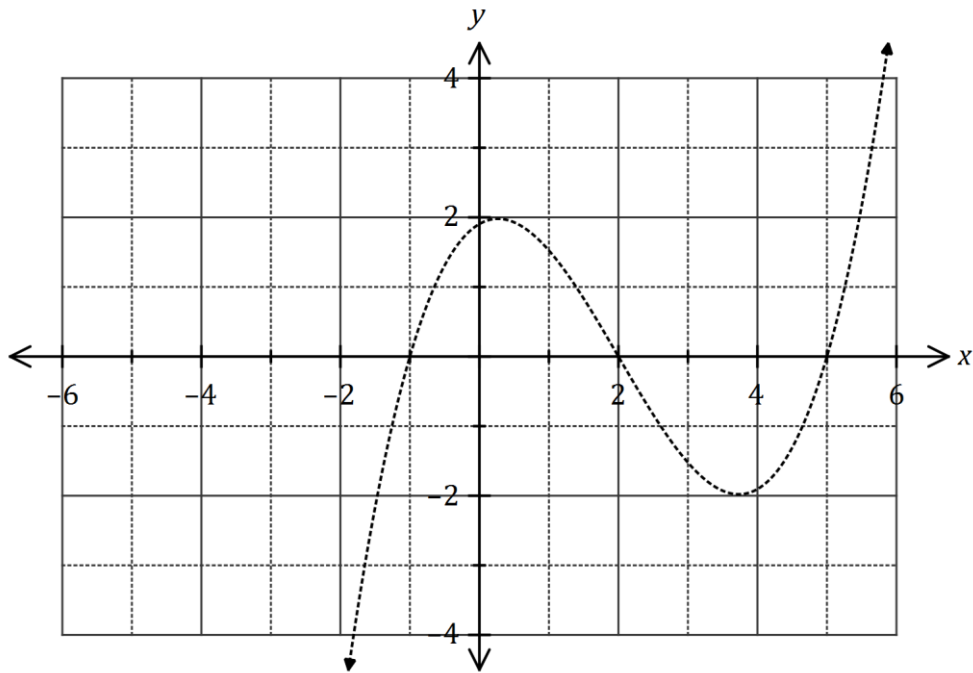The graph of $y = f(x)$ is shown below.

(a)   Sketch the graph of $y = f(|x|)$ on the axes below. (2 marks)
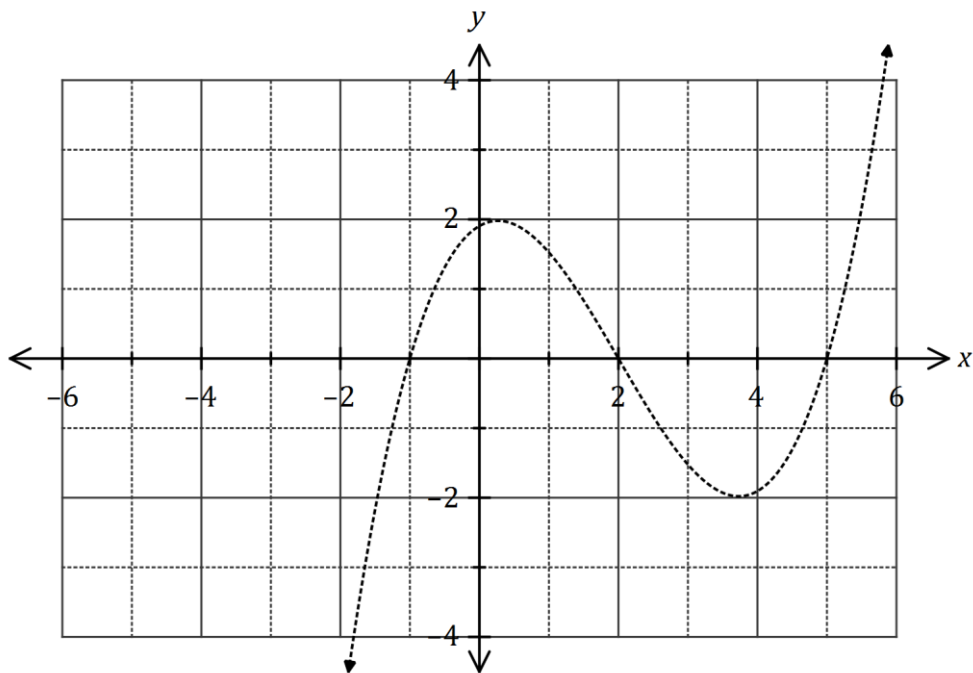
(b)   Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below.   (4 marks)

(c)   Sketch the graph of $y = |f(|x|)|$ on the axes below.   (2 marks)

Question 16 **(8 marks)**

(a) Let $r \operatorname{cis} \theta$ be a point in the complex plane. Determine, in terms of r and θ, the polar form of this point after it is rotated by $\frac{\pi}{4}$ about the origin and then reflected in the real axis.
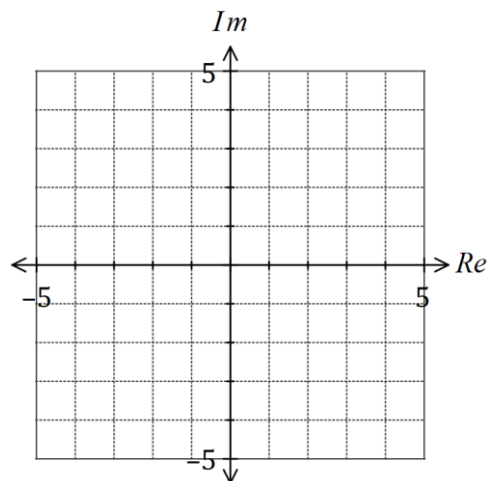
(2 marks)

(b) Let $f(w) = -i\overline{w} + 1 + i$.

(i) Complete the following table. (3 marks)

w	$1 + 2i$	$-3 + i$	$-1 - 4i$
$f(w)$			

(ii) Sketch each point, w, and join it with a dotted line to its image, $f(w)$, on the diagram below. (1 mark)

(iii) Describe the geometric transformation that $f(w)$ represents. (2 marks)

Question 17 **(8 marks)**

The velocity vector of a small body at time t seconds is $\mathbf{v}(t) = 4\sin(4t)\,\mathbf{i} + 3\cos(4t)\,\mathbf{j}$ ms^{-1}.
Initially, the body has position vector $2\mathbf{i} + 3\mathbf{j}$.

(a)   Determine the acceleration vector for the body when $t = \dfrac{5\pi}{16}$.   (2 marks)

(b)   Show that the minimum speed of the body is 3 ms^{-1}.   (3 marks)

(c)   Determine the distance the body travels between $t = 0$ and the first instant after this time that the body returns to its initial position, rounding your answer to the nearest cm.

    (3 marks)

Question 18 **(8 marks)**

Points A, B and C have position vectors $(a, 0, 0), (0, b, 0)$ and $(0, 0, c)$ respectively, where a, b and c are non-zero, real constants. Point M is the midpoint of B and C.

(a)   Sketch a vector diagram representing this situation. (1 mark)

(b)   Use a vector method to prove that $\overrightarrow{AM}$ is perpendicular to $\overrightarrow{BC}$ when $|\overrightarrow{OB}| = |\overrightarrow{OC}|$.

(7 marks)

Question 19 **(8 marks)**

Functions f and g are defined as $f(x) = x^2 + ax + 2a$ and $g(x) = \dfrac{x}{x+b}$, where a and b are constants.

(a)   Let $a = -4$ and $b = 3$.

    (i)   State, with reasons, whether the composite function $f(g(x))$ is a one-to-one function over its natural domain.

             (2 marks)

    (ii)   Determine any domain restrictions required so that the composite function $g(f(x))$ is defined. (3 marks)

(b)   Determine the relationship between a and b so that the composite function $g(f(x))$ is always defined for $x \in \mathbb{R}$. (3 marks)

See next page

Question 20   **(8 marks)**

The position vector of a boat motoring on a lake is given by $\mathbf{r}(t) = (6 - 6\cos^2(t))\mathbf{i} + 4\sin(2t)\,\mathbf{j}$, where t is the time, in hours, after it leaves (0, 0) and distances are in kilometres. The path of the boat is shown below, where the shoreline is represented by the line $x = 0$.
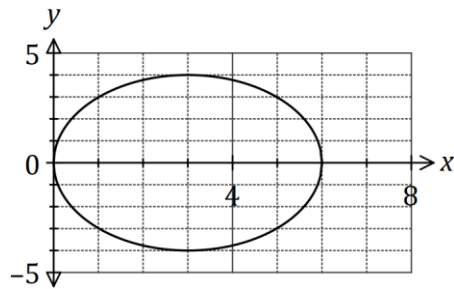
(a)   Express the path of the particle as a Cartesian equation.   (3 marks)

(b)   On the graph above, mark the position of the boat when it is first 4.5 km from the shoreline and indicate the direction it is travelling.   (1 mark)

(c)   Determine the speed of the boat when it is first 4.5 km from the shoreline.   (4 marks)

Question 21   **(7 marks)**

(a)   Consider the complex equation $z^5 = 4 - 4i$.

Solve the equation, giving all solutions in the form $r\operatorname{cis}\theta$ where $r > 0$ and $-\pi < \theta \leq \pi$.
(4 marks)

(b)   One solution to the complex equation $z^5 = -9\sqrt{3}$ is $z = \sqrt{3}\operatorname{cis}\left(\frac{3\pi}{5}\right)$.

Let u be the solution to $z^5 = -9\sqrt{3}$ so that $-\frac{\pi}{2} \leq \arg(u) \leq 0$. Determine $\arg(u - \sqrt{3}i)$ in exact form.
(3 marks)

Supplementary page

Question number: _____

Supplementary page

Question number: _____